is often of interest for a competing risks data analysis.

There is rich literature which evaluates direct covariate e ects on cumulative incidence for clustered competing risks data. Scheike et al. (2010) studied a semiparametric random e ects model based on the direct binomial modeling, where the marginal cumulative incidence functions follow a generalized semiparametric additive model. Logan et al. (2011) proposed a pseudovalue approach based on the jackknife estimator and a generalized estimating equation. Ruan and Gray (2008) proposed a non-parametric multiple imputation method. Zhou et al. (2012) extended the proportional subdistribution hazards (PSH) model of Fine and Gray (1999) to clustered competing risks data. However, the asymptotic results of all of these methods are limited to covariate-independent censoring. And Zhou et al. (2012) is limited to the PSH structure. In practice, the censoring distribution may depend on some covariates and the PSH assumption may not hold. Addressing these limitations under the strati ed PSH model is crucial in practice because many clinicians and investigators widely use the PSH model of Fine and Gray (1999) for competing risks data analysis.

Therefore, we study a strati ed PSH model with covariate-adjusted censoring weight for clustered data in this article. The survival probability of censoring is estimated using the marginal strati ed proportional hazards model. Inay InH()-4305415(PSH)]TJ 0 -12.96(co)28(

and the at-risk indicator associated with censoring, respectively. Let $\xi_{ij} = 1$ if the j^{th} stratum has at least one subject from cluster i; otherwise $\xi_{ij} = 0$.

We assume the censoring distribution follows the stratified proportional hazards model (Cox, 1972):

$$\lambda_{Cj}(t; Z_{ijk}) = \lambda_{C0j}(t)\exp(\alpha_0^{\intercal} Z_{ijk}); \; j = 1, \ldots, S; \tag{1}$$

where $\lambda_{C0j}(t)$ is an unspecified baseline hazard function and α_0 is the true parameter vector for censoring. Define

$$S^{(r)}_{Cj}(\alpha; t) = \frac{1}{n_j}\sum_{i=1}^{n} \xi_{ij} \sum_{k=1}^{n_{ij}} Y^{C}_{ijk}(t) Z_{ijk}^{\otimes r} \exp(\alpha^{\intercal} Z_{ijk}); \; j = 1, \ldots, S;$$

for $r = 0, 1, 2$ such that $a^{\otimes 0} = 1$, $a^{\otimes 1} = a$ and $a^{\otimes 2} = aa^{\intercal}$ for a. The score function for α is

$$U_C(\alpha) = \sum_{i=1}^{n}\sum_{j=1}^{S} \xi_{ij} \sum_{k=1}^{n_{ij}} \int_0^{\infty} \left\{ Z_{ijk} - \frac{S^{(1)}_{Cj}(\alpha; u)}{S^{(0)}_{Cj}(\alpha; u)} \right\} dN^{C}_{ijk}(u) = 0. \tag{2}$$

Let $\hat{\alpha}$ be the estimator for α_0 from (2). The Breslow-type estimator of $\Lambda_{C0j}(t) = \int_0^t \lambda_{C0j}(t)$ is

$$\hat{\Lambda}_{C0j}(t) = \frac{1}{n_j}\sum_{i=1}^{n} \xi_{ij} \sum_{k=1}^{n_{ij}} \int_0^t \frac{dN^{C}_{ijk}(s)}{S^{(0)}_{Cj}(\hat{\alpha}; s)}.$$

Thus, the estimated survival probability of the censoring distribution is $\hat{G}_{Cj}(t \mid Z_{ijk}) = \exp\{-\hat{\Lambda}_{C0j}(t)\exp(\hat{\alpha}^{\intercal} Z_{ijk})\}$.

The covariate-adjusted censoring weight function is $\hat{w}^{COX}_{ijk}(t) = I(C_{ijk} \geq X_{ijk} \wedge t)\hat{G}_{C;j}(t; Z_{ijk})/\hat{G}_{C;j}(X_{ijk} \wedge t; Z_{ijk})$. For $j = 1, \ldots, S$; define

$$S^{(r)}_{COX;j}(\beta; t) = \frac{1}{n_j}\sum_{i=1}^{n} \xi_{ij} \sum_{k=1}^{n_{ij}} \hat{w}^{COX}_{ijk}(t) Y^{1}_{ijk}(t) Z_{ijk}^{\otimes r} \exp(\beta^{\intercal} Z_{ijk}).$$

Our main interest is to evaluate the direct effects of covariates on the cumulative incidence function of Cause 1, $F_{1j}(t \mid Z_{ijk}) = P(T_{ijk} \leq t, \epsilon_{ijk} = 1 \mid Z_{ijk})$. The stratified PSH model for Cause 1 is

$$\lambda_{1j}(t \mid Z_{ijk}) = \lambda_{10j}(t)\exp(\beta_0^{\intercal} Z_{ijk}); \; j = 1, \ldots, S; \tag{3}$$

where $\lambda_{10j}(t)$ is an unspecified baseline subdistribution hazard function and β_0 is the true parameter vector. We estimate β_0 in (3) by setting the following score equation equal to zero:

$$U_{COX}(\beta) = \sum_{i=1}^{n}\sum_{j=1}^{S}\sum_{k=1}^{n_{ij}} \int_0^{\tau} \left\{ Z_{ijk} - \frac{S^{(1)}_{COX;j}(\beta;u)}{S^{(0)}_{COX;j}(\beta;u)} \right\} \hat{w}^{COX}_{ijk}(u)\,dN^1_{ijk}(u) . \tag{4}$$

Denote the estimator of β_0 as $\hat{\beta}$. The Breslow-type estimator of the cumulative baseline subdistribution hazard function of Cause 1, $\Lambda_{10j}(t) = \int_0^t \lambda_{10j}(s)$, for stratum j is

$$\hat{\Lambda}_{10j}(t) = \frac{1}{n_j}\int_0^t \frac{1}{S^{(0)}_{COX;j}(\hat{\beta};s)} \sum_{i=1}^{n}\sum_{k=1}^{n_{ij}} \hat{w}^{COX}_{ijk}(s)\,dN^1_{ijk}(s).$$

3. Asymptotic Properties

We establish the asymptotic properties of the proposed estimators in this section. Define $M^1_{ijk}(t) = N^1_{ijk}(t) - \int_0^t Y^1_{ijk}(u)e^{\beta_0^T Z_{ijk}}\,d\Lambda_{10j}(u)$ and $M^C_{ijk}(t) = N^C_{ijk}(t) - \int_0^t Y_{ijk}(u)e^{\gamma^T Z_{ijk}}\,d\Lambda_{C0j}(u)$ for Cause 1 and censoring processes, respectively. We assume the following conditions:

(A1) $P(T_{ijk} > \tau) > 0$ and $P(C_{ijk} > \tau) > 0$ for all i, j and k.
(A2) The covariates Z are bounded almost surely.
(A3) $\int_0^{\tau} \lambda_{C0j}(t)dt < \infty$ for $j = 1, \ldots, S$.
(A4) $-1/n\ \partial U_C(\gamma_0)/\partial \gamma_0^T$ converges to $\Sigma_C(\gamma_0)$ which is positive definite.
(A5) $\int_0^{\tau} \lambda_{10j}(t)dt < \infty$ for $j = 1, \ldots, S$.
(A6) $-1/n\ \partial U_{COX}(\beta_0)/\partial \beta_0^T$ converges to $\Omega(\beta_0)$ which is positive definite.
(A7) There exists a neighborhood B_C of γ_0 and $s^{(r)}_{Cj}$ defined on $B_C \times [0, \tau]$ such

$n^{1/2}(\hat{b} \quad {}_0)$ converges in distribution to a zero-mean normal distribution with covariance matrix $\;({}_0)^{\;1} \quad ({}_0)^{\;1}$, where

$$= E\left\{ {}^{COX}_{1::} + {}^{COX}_{1::} \right\}^{\otimes 2};$$

$$ {}^{COX}_{i::} = \sum_{j=1}^{S} {}_{ij} \sum_{k=1}^{n_{ij}} \int_0 \left\{ Z_{ijk} \quad \frac{s^{(1)}_{COX;j}(\;{}_0;u)}{s^{(0)}_{COX;j}(\;{}_0;u)} \right\} w^{COX}_{ijk}(u)\,dM^{1}_{ijk}(u);$$

$$ {}^{COX}_{i::} = \sum_{j=1}^{S} {}_{ij} \sum_{k=1}^{n_{ij}} \int_0 q^{(1)}_{ijk}(u)\,dM^{C}_{ijk}(u):$$

The expression for $q^{(1)}_{ijk}(u)$ and its proof are provided in the Appendix is provided in the Supplementary Materials. We consider the same strata for events from Cause 1 and censoring for a mathematical simplicity in Theorem 3.1. This avoids the abuse of complicated notations and subscripts. However, similarly to Kim et al. (2020), one can show the asymptotic results of Theorem 3.1 even when strata for models for Cause 1

The asymptotic variances of the proposed estimators for and ${}_{10j}(t)$ when using Kaplan-Meier estimates for censoring are provided in the Supplementary Materials.

4. Simulation

We conduct simulation studies using R package **adjSURVCI** (Khanal and Ahn, 2021). We consider two weight functions: covariate-dependent weight $\hat{w}_{ijk}^{COX}(t)$ and covariate-independent weight $\hat{w}_{ijk}^{KM}(t) = I(C_{ijk} \geq X_{ijk} \wedge t)\hat{G}_{C;j}(t)/\hat{G}_{C;j}(X_{ijk} \wedge t)$, where $\hat{G}_{C;j}(\cdot)$ is a Kaplan-Meier estimator for the survival probability of censoring for stratum j. We consider two competing risks with Cause 1 being the main event of interest and 2 strata. Similarly to Logan et al. (2011), we generate clustered competing risks data for Causes 1, 2, and censoring for stratum j using

$$F_{1j}(t \mid \omega; Z) = 1 - [1 - p(1 - \exp\{ {}_j t\})]$$

where MVN is the multivariate normal distribution. However, the middle 50% of the clusters contain 4 observations. For those clusters, we assume

$$W_i \sim MVN\left(0, \begin{bmatrix} 1 & 0.5 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 1 \end{bmatrix}\right).$$

The number of strata for censoring is 2 and ${}_0 = (2.5, 2.5, \; 3)^{\prime}$. Table 7 presents the other parameter values for the AFT model. With these values, it results in approximately 30% censoring, 40% Cause 1, and 30% Cause 2. Table 8 provides the simulation results. The proposed method with $\hat{w}_{ijk}^{COX}(t)$ has little bias and the coverage rates close to 95%. The estimates of the proposed method with $\hat{w}_{ijk}^{KM}(t)$ are biased and their coverage rates are less than 95%. The proposed method with $\hat{w}_{ijk}^{COX}(t)$ is robust against mis-specified modeling of censoring under this limited setting.

5. Real data analysis

We apply our method to the data set of Pidala et al. (2014) to investigate the impact of donor-recipient HLA matching on chronic GVHD of acute lymphoblastic leukemia patients who received unrelated donor allogeneic hematopoietic cell transplantation, where death without experiencing chronic GVHD is a competing event. The variables considered in this application are GVHD prophylaxis (FK506 + (MTX or MMF or steroids) + other , FK506 + other, CsA + MTX + other, CsA + other(No MTX), T-cell depletion and Other); HLA matching (8/8, 7/8 and 6/8); year of transplant (1999–2002, 2003–2006 and 2007–2011); graft type (bone marrow and peripheral blood); in vivo T-cell depletion (no and yes) and total body irradiation (no and yes). The total sample size is 2200. Table 9 shows patient characteristics for the 2200 patients. There are 451 censored patients, 879 patients experienced chronic GVHD and 870 patients died without experiencing chronic GVHD.

First, we check the proportional hazard assumption for censoring by examining the coefficient of variable $\times \log(t)$ under the marginal PH model for each variable. Year of transplant does not satisfy the proportional hazard assumption with p-value < 0.001. Therefore, we stratify year of transplant for the proportional hazards model for censoring. We also include the other variables in Table 9 in modeling the censoring distribution. Next, we check the PSH assumption for each variable by examining the coefficient of variable $\times \log(t)$ based on the proposed model. GVHD prophylaxis does not satisfy the PSH assumption with p-value < 0.001. Hence, we stratify it for the PSH model for chronic GVHD. We also check a study center effect using the score test of homogeneity (Commenges and Andersen, 1995) and it was significant with p-value < 0.001. The level of significance is set at 0.05. Then, we fit four different stratified PSH model as follows:

(1) Proposed method with $\hat{w}_{ijk}^{COX}(t)$;
(2) Proposed method with $\hat{w}_{ijk}^{COX}(t)$, but ignores clusters and instead treat the data as independent data;
(3) Proposed method with $\hat{w}_{ijk}^{KM}(t)$.
(4) Proposed method with $\hat{w}_{ijk}^{KM}(t)$, but ignores clusters and instead treat the data as independent data;

Table 10 show the analysis results. The results for the models with [illegible]

Table 2. Simulation results for parameter estimation for Cause 1

				$\hat{w}_{ijk}^{COX}(t)$			$\hat{w}_{ijk}^{KM}(t)$		
Scenario		n	0	Bias	SE(SD)	CP	Bias	SE(SD)	CP
CDC	0.25	200	01	-0.003	0.089(0.090)	0.946	0.051	0.086(0.089)	0.894
			02	-0.001	0.356(0.356)	0.950	0.075	0.355(0.356)	0.944
			03	-0.015	0.209(0.211)	0.950	-0.050	0.207(0.213)	0.945
		400	01	-0.001	0.063(0.063)	0.945	0.054	0.061(0.062)	0.849
			02	0.003	0.252(0.252)	0.948	0.079	0.251(0.253)	0.939
			03	-0.007	0.146(0.147)	0.951	-0.043	0.146(0.148)	0.944
		800	01	0.000	0.044(0.044)	0.952	0.056	0.043(0.043)	0.732
			02	0.004	0.178(0.180)	0.947	0.081	0.178(0.180)	0.922
			03	-0.002	0.103(0.102)	0.950	-0.039	0.103(0.102)	0.939
	0.5	200	01	0.000	0.087(0.088)	0.943	0.111	0.079(0.082)	0.697
			02	-0.005	0.332(0.336)	0.942	0.140	0.328(0.334)	0.927
			03	-0.016	0.210(0.216)	0.943	-0.168	0.209(0.219)	0.885
		400	01	0.001	0.061(0.061)	0.950	0.114	0.056(0.056)	0.464
			02	0.005	0.234(0.235)	0.948	0.151	0.232(0.233)	0.899
			03	-0.007	0.147(0.150)	0.944	-0.160	0.148(0.152)	0.827
		800	01	0.000	0.043(0.043)	0.951	0.114	0.040(0.040)	0.190
			02	0.003	0.165(0.165)	0.949	0.152	0.164(0.164)	0.853
			03	-0.003	0.103(0.104)	0.949	-0.157	0.104(0.105)	0.685
	1	200	01	0.002	0.085(0.085)	0.948	0.171	0.073(0.074)	0.357
			02	0.007	0.239(0.237)	0.951	0.203	0.229(0.230)	0.858
			03	-0.021	0.211(0.206)	0.958	-0.445	0.207(0.207)	0.421
		400	01	-0.001	0.060(0.059)	0.950	0.170	0.051(0.052)	0.094
			02	0.006	0.169(0.167)	0.951	0.205	0.162(0.162)	0.755
			03	-0.007	0.147(0.149)	0.948	-0.435	0.146(0.149)	0.134
		800	01	0.001	0.042(0.042)	0.951	0.173	0.036(0.037)	0.005
			02	0.001	0.119(0.120)	0.951	0.201	0.114(0.115)	0.580
			03	-0.003	0.103(0.103)	0.948	-0.431	0.103(0.103)	0.009
CIC	0.25	200	01	-0.004	0.074(0.076)	0.943	-0.004	0.074(0.077)	0.941
			02	-0.003	0.367(0.370)	0.951	-0.002	0.367(0.371)	0.950
			03	-0.005	0.157(0.158)	0.948	-0.006	0.156(0.158)	0.947
		400	01	-0.002	0.052(0.053)	0.948	-0.002	0.052(0.053)	0.945
			02	0.002	0.259(0.259)	0.950	0.002	0.259(0.260)	0.949
			03	-0.003	0.110(0.111)	0.950	-0.004	0.110(0.111)	0.951
		800	01	0.000	0.037(0.037)	0.953	0.000	0.037(0.037)	0.954
			02	0.005	0.183(0.186)	0.944	0.005	0.183(0.186)	0.945
			03	-0.001	0.078(0.076)	0.954	-0.001	0.078(0.077)	0.953
	0.5	200	01	-0.004	0.070(0.072)	0.946	-0.004	0.070(0.072)	0.944
			02	-0.007	0.328(0.330)	0.947	-0.007	0.329(0.330)	0.948
			03	-0.006	0.151(0.154)	0.946	-0.007	0.151(0.156)	0.943
		400	01	-0.001	0.049(0.050)	0.946	-0.001	0.050(0.050)	0.945
			02	0.000	0.231(0.232)	0.946	0.000	0.232(0.233)	0.947
			03	-0.003	0.106(0.107)	0.950	-0.003	0.107(0.108)	0.947
		800	01	0.000	0.035(0.035)	0.951	0.000	0.035(0.035)	0.949
			02	0.002	0.163(0.163)	0.953	0.002	0.164(0.163)	0.953
			03	-0.001	0.075(0.075)	0.948	-0.002	0.075(0.075)	0.947
	1	200	01	-0.003	0.066(0.066)	0.948	-0.003	0.066(0.066)	0.946
			02	0.002	0.230(0.237)	0.939	0.002	0.231(0.239)	0.939
			03	-0.007	0.150(0.149)	0.952	-0.008	0.150(0.150)	0.950
		400	01	-0.002	0.047(0.047)	0.946	-0.002	0.047(0.048)	0.942
			02	0.004	0.163(0.164)	0.945	0.004	0.164(0.167)	0.945
			03	-0.004	0.105(0.107)	0.946	-0.004	0.106(0.108)	0.944
		800	01	0.000	0.033(0.033)	0.952	0.000	0.033(0.033)	0.950
			02	-0.001	0.115(0.116)	0.946	-0.002	0.116(0.118)	0.947
			03	-0.001	0.074(0.075)	0.949	-0.001	0.075(0.076)	0.951

Table 3. Simulation results for cumulative baseline subdistribution hazard estimation at t = 0:1 and 0:7

Scenario		n	t	Stratum	$\hat{\Lambda}_{ijk}^{COX}(t)$			$\hat{\Lambda}_{ijk}^{KM}(t)$		
					Bias	SE(SD)	CP	Bias	SE(SD)	CP
CDC	0.25	200	0.1	0	-0.007	0.126(0.128)	0.932	-0.001	0.124(0.127)	0.924
				1	-0.007	0.144(0.144)	0.940	-0.001	0.141(0.143)	0.929
			0.7	0	-0.011	0.190(0.190)	0.938	0.008	0.183(0.187)	0.921
				1	-0.012	0.209(0.209)	0.938	0.008	0.202(0.205)	0.924
		400	0.1	0	-0.005	0.088(0.090)	0.940	0.002	0.087(0.089)	0.929
				1	-0.005	0.101(0.103)	0.939	0.002	0.100(0.103)	0.929
			0.7	0	-0.008	0.133(0.135)	0.941	0.013	0.129(0.132)	0.924
				1	-0.008	0.146(0.149)	0.943	0.013	0.142(0.145)	0.930
		800	0.1	0	-0.003	0.062(0.063)	0.942	0.004	0.061(0.062)	0.933
				1	-0.003	0.071(0.072)	0.943	0.003	0.071(0.071)	0.936
			0.7	0	-0.005	0.093(0.094)	0.945	0.016	0.091(0.092)	0.928
				1	-0.005	0.103(0.104)	0.946	0.015	0.100(0.102)	0.931
	0.5	200	0.1	0	-0.001	0.064(0.067)	0.913	0.016	0.060(0.064)	0.865
				1	-0.002	0.084(0.087)	0.920	0.019	0.080(0.083)	0.880
			0.7	0	-0.004	0.145(0.150)	0.923	0.052	0.131(0.138)	0.852
				1	-0.003	0.178(0.184)	0.929	0.061	0.162(0.169)	0.857
		400	0.1	0	-0.001	0.046(0.047)	0.934	0.016	0.043(0.044)	0.881
				1	-0.002	0.060(0.061)	0.936	0.019	0.057(0.058)	0.890
			0.7	0	-0.005	0.102(0.103)	0.938	0.053	0.093(0.095)	0.851
				1	-0.005	0.125(0.127)	0.937	0.061	0.115(0.117)	0.858
		800	0.1	0	-0.001	0.032(0.033)	0.942	0.017	0.030(0.031)	0.870
				1	-0.001	0.042(0.043)	0.941	0.020	0.040(0.041)	0.880
			0.7	0	-0.003	0.072(0.073)	0.943	0.055	0.066(0.066)	0.817
				1	-0.004	0.089(0.090)	0.942	0.062	0.081(0.082)	0.834
	1	200	0.1	0	0.000	0.018(0.019)	0.913	0.014	0.015(0.015)	0.730
				1	0.000	0.030(0.030)	0.930	0.026	0.025(0.025)	0.715
			0.7	0	-0.001	0.088(0.088)	0.937	0.099	0.065(0.066)	0.592
				1	-0.001	0.136(0.136)	0.938	0.156	0.103(0.104)	0.598
		400	0.1	0	0.000	0.013(0.013)	0.938	0.014	0.010(0.011)	0.654
				1	-0.001	0.021(0.022)	0.938	0.025	0.017(0.018)	0.632
			0.7	0	-0.003	0.062(0.064)	0.938	0.098	0.046(0.048)	0.424
				1	-0.004	0.096(0.099)	0.938	0.155	0.073(0.075)	0.423
		800	0.1	0	0.000	0.009(0.0Td [(1)-2653(-0.9316600.075))-1095(0.423)]TJ 1.030)				
				1	0.000	0.015(0.015)	09480	0.026	0.142(0.122)	04382
			0.7	0	-0.001	0.0423				
				1	-0.001	0.017(0.067)	0.438	0.198	0.521(0.541)	01682

Table 4. Simulation results for parameter estimation when ignoring clusters

				$\widehat{w}_{ijk}^{COX}(t)$		$\widehat{w}_{ijk}^{KM}(t)$	
Scenario		n	0	SE(SD)	CP	SE(SD)	CP
CDC	0.25	200	01	0.083(0.090)	0.931	0.081(0.089)	0.874
			02	0.230(0.356)	0.796	0.230(0.356)	0.790
			03	0.204(0.211)	0.947	0.201(0.213)	0.943
		400	01	0.058(0.063)	0.930	0.057(0.062)	0.818
			02	0.162(0.252)	0.792	0.162(0.253)	0.773
			03	0.142(0.147)	0.944	0.141(0.148)	0.939
		800	01	0.041(0.044)	0.932	0.040(0.043)	0.697
			02	0.114(0.180)	0.786	0.114(0.180)	0.750
			03	0.100(0.102)	0.945	0.099(0.102)	0.932
	0.5	200	01	0.084(0.088)	0.933	0.077(0.082)	0.675
			02	0.232(0.336)	0.827	0.228(0.334)	0.776
			03	0.208(0.216)	0.944	0.205(0.219)	0.881
		400	01	0.059(0.061)	0.938	0.054(0.056)	0.441
			02	0.163(0.235)	0.823	0.160(0.233)	0.739
			03	0.145(0.150)	0.944	0.144(0.152)	0.817
		800	01	0.041(0.043)	0.940	0.038(0.040)	0.173
			02	0.115(0.165)	0.832	0.113(0.164)	0.652
			03	0.101(0.104)	0.945	0.101(0.105)	0.670
	1	200	01	0.085(0.085)	0.947	0.073(0.074)	0.356
			02	0.232(0.237)	0.944	0.222(0.230)	0.848
			03	0.211(0.206)	0.958	0.207(0.207)	0.422
		400	01	0.060(0.059)	0.951	0.051(0.052)	0.092
			02	0.163(0.167)	0.944	0.157(0.162)	0.736
			03	0.147(0.149)	0.950	0.145(0.149)	0.132
		800	01	0.042(0.042)	0.950	0.036(0.037)	0.005
			02	0.115(0.120)	0.944	0.110(0.115)	0.552
			03	0.103(0.103)	0.947	0.102(0.103)	0.009
CIC	0.25	200	01	0.067(0.076)	0.915	0.067(0.077)	0.913
			02	0.225(0.370)	0.771	0.225(0.371)	0.771
			03	0.154(0.158)	0.947	0.153(0.158)	0.946
		400	01	0.047(0.053)	0.920	0.047(0.053)	0.918
			02	0.158(0.259)	0.769	0.158(0.260)	0.770
			03	0.108(0.111)	0.946	0.108(0.111)	0.946
		800	01	0.033(0.037)	0.930	0.033(0.037)	0.929
			02	0.111(0.186)	0.755	0.112(0.186)	0.759
			03	0.076(0.076)	0.949	0.076(0.077)	0.948
	0.5	200	01	0.066(0.072)	0.929	0.066(0.072)	0.929
			02	0.219(0.330)	0.806	0.220(0.330)	0.807
			03	0.149(0.154)	0.945	0.150(0.156)	0.944
		400	01	0.046(0.050)	0.929	0.046(0.050)	0.929
			02	0.153(0.232)	0.806	0.154(0.233)	0.811
			03	0.105(0.107)	0.948	0.105(0.108)	0.944
		800	01	0.033(0.035)	0.934	0.033(0.035)	0.937
			02	0.108(0.163)	0.803	0.109(0.163)	0.802
			03	0.074(0.075)	0.9451	0.51 0.51	

8091 Tf -27.092 -12.F31 7.9701 Tf 6.17 -1.636 Td [(02)]TJ/F28 10.9091 T91 T

Table 5. Simulation results for cumulative baseline subdistribution hazard estimation at t = 0:1 and 0:7 when ignoring clusters

					$\hat{\Lambda}_{ijk}^{COX}(t)$		$\hat{\Lambda}_{ijk}^{KM}(t)$	
Scenario		n	t	Stratum	SE(SD)	CP	SE(SD)	CP
CDC	0.25	200	0.1	0	0.103(0.128)	0.881	0.100(0.127)	0.870
				1	0.118(0.144)	0.887	0.115(0.143)	0.880
			0.7	0	0.157(0.190)	0.892	0.151(0.187)	0.874
				1	0.173(0.209)	0.891	0.167(0.205)	0.877
		400	0.1	0	0.072(0.090)	0.880	0.071(0.089)	0.871
				1	0.082(0.103)	0.878	0.081(0.103)	0.868
			0.7	0	0.110(0.135)	0.891	0.106(0.132)	0.868
				1	0.121(0.149)	0.889	0.117(0.145)	0.869
		800	0.1	0	0.050(0.063)	0.887	0.050(0.062)	0.876
				1	0.058(0.072)	0.886	0.057(0.071)	0.879
			0.7	0	0.077(0.094)	0.893	0.075(0.092)	0.871
				1	0.085(0.104)	0.890	0.083(0.102)	0.872
	0.5	200	0.1	0	0.054(0.067)	0.870	0.050(0.064)	0.819
				1	0.071(0.087)	0.882	0.067(0.083)	0.833
			0.7	0	0.125(0.150)	0.886	0.112(0.138)	0.806
				1	0.154(0.184)	0.889	0.139(0.169)	0.814
		400	0.1	0	0.038(0.047)	0.889	0.036(0.044)	0.825
				1	0.050(0.061)	0.891	0.047(0.058)	0.830
			0.7	0	0.088(0.103)	0.898	0.079(0.095)	0.796
				1	0.108(0.127)	0.899	0.098(0.117)	0.809
		800	0.1	0	0.027(0.033)	0.889	0.025(0.031)	0.801
				1	0.035(0.043)	0.896	0.033(0.041)	0.818
			0.7	0	0.062(0.073)	0.897	0.056(0.066)	0.744
				1	0.076(0.090)	0.901	0.069(0.082)	0.763
	1	200	0.1	0	0.018(0.019)	0.914	0.014(0.015)	0.727
				1	0.030(0.030)	0.931	0.024(0.025)	0.711
			0.7	0	0.087(0.088)	0.936	0.064(0.066)	0.586
				1	0.135(0.136)	0.938	0.102(0.104)	0.594
		400	0.1	0	0.013(0.013)	0.936	0.010(0.011)	0.646
				1	0.021(0.021)	0.936	0.017(0.018)	0.628
			0.7	0	0.062(0.064)	0.935	0.045(0.048)	0.413
				1	0.096(0.099)	0.937	0.072(0.075)	0.420
		800	0.1	0	0.009(0.009)	0.946	0.007(0.007)	0.497
				1	0.015(0.015)	0.947	0.012(0.012)	0.428
			0.7	0	0.043(0.044)	0.944	0.033(0.032)	0.171
				1	0.067(0.067)	0.946	0.051(0.051)	0.164
CIC	0.25	200	0.1	0	0.074(0.100)	0.851	0.074(0.100)	0.849
				1	0.084(0.113)	0.858	0.084(0.113)	0.858
			0.7	0	0.111(0.148)	0.860	0.111(0.148)	0.861
				1	0.120(0.161)	0.858	0.120(0.161)	0.858
		400	0.1	0	0.052(0.070)	0.850	0.052(0.070)	0.850
				1	0.059(0.079)	0.858	0.059(0.079)	0.858
			0.7	0	0.078(0.103)	0.863	0.078(0.103)	0.862
				1	0.084(0.112)	0.864	0.084(0.112)	0.864
		800	0.1	0	0.037(0.049)	0.857	0.037(0.049)	0.859
				1	0.042(0.057)	0.854	0.042(0.057)	0.853
			0.7	0	0.055(0.072)	0.864	0.055(0.072)	0.864
				1	0.059(0.079)	0.861	0.059(0.079)	0.860
	0.5	200	0.1	0	0.043(0.056)	0.860	0.043(0.056)	0.858
				1	0.056(0.073)	0.866	0.056(0.073)	0.863
			0.7	0	0.098(0.124)	0.873	0.098(0.124)	0.871
				1	0.117(0.148)	0.876	0.117(0.148)	0.873
		400	0.1	0	0.030(0.039)	0.873	0.030(0.039)	0.876
				1	0.040(0.051)	0.872	0.040(0.051)	0.875
			0.7	0	0.069(0.086)	0.881	0.069(0.086)	0.876
				1	0.083(0.104)	0.880	0.083(0.104)	0.879
		800	0.1	0	0.021(0.027)	0.877	0.022(0.027)	0.879
				1	0.028(0.036)	0.873	0.028(0.036)	0.873
			0.7	0	0.049(0.060)	0.884	0.049(0.060)	0.885
				1	0.058(0.073)	0.884	0.058(0.073)	0.883
	1	200	0.1	0	0.015(0.016)	0.917	0.015(0.016)	0.919
				1	0.025(0.025)	0.928	0.025(0.026)	0.929
			0.7	0	0.069(0.071)	0.933	0.069(0.071)	0.932
				1	0.102(0.104)	0.944	0.102(0.104)	0.942
		400	0.1	0	0.011(0.011)	0.936	0.011(0.011)	0.933
				1	0.018(0.018)	0.937	0.018(0.018)	0.938
			0.7	0	0.049(0.050)	0.940	0.049(0.050)	0.938
				1	0.072(0.075)	0.940	0.072(0.075)	0.937
		800	0.1	0	0.008(0.008)	0.943	0.008(0.008)	0.943
				1	0.012(0.012)	0.944	0.012(0.012)	0.947
			0.7	0	0.035(0.035)	0.945	0.035(0.035)	0.944
				1	0.051(0.052)	0.943	0.051(0.052)	0.942

Table 6. Simulation results for parameter estimates of Cause 1, when strata for Cause 1 and censoring models are di erent

Table 8. Simulation results of parameter estimates for Cause 1 when censoring follows AFT model

		$\hat{w}_{ijk}^{COX}(t)$			$\hat{w}_{ijk}^{KM}(t)$		
n	$_0$	Bias	SE(SD)	CP	Bias	SE(SD)	CP

Table 9. Patient characteristics

	8/8	7/8	6/8	Total
Year of transplant				
1999 - 2002	237	131	64	432
2003 - 2006	497	223	60	780
2007 - 2011	702	259	27	988
Graft type				
Peripheral blood	759	297	65	1121
Bone marrow	677	316	86	1079
In vivo T cell depletion				
No	1087	400	92	1579
Yes	349	213	59	621
Total body irradiation				
No	166	69	4	239
Yes	1270	544	147	1961
GVHD prophylaxis				
FK506 + (MTX or MMF or steroids) + other	779	300	58	1137
FK506 + other	85	21	4	110
CsA + MTX + other	407	187	50	644
CsA + other(No MTX)	48	23	6	77
T-cell depletion	95	63	31	189
Other	22	19	2	43

Table 10.: Real data analysis

	$\widehat{w}_{ijk}^{COX}(t)$ accounting for clusters		$\widehat{w}_{ijk}^{COX}(t)$ ignoring clusters		$\widehat{w}_{ijk}^{KM}(t)$ accounting for clusters		$\widehat{w}_{ijk}^{KM}(t)$ ignoring clusters	
	Estimate (SE)	p-value	Estimate (SE)	p-value	Estimate (SE)	p-value	Estimate (SE)	p-value
HLA matching								
8/8 (ref)	-	0.382	-	0.413	-	0.384	-	0.411
7/8	0.069 (0.077)	0.369	0.068 (0.078)	0.380	0.068 (0.076)	0.371	0.068 (0.077)	0.379
6/8	-0.133 (0.224)	0.554	-0.133 (0.157)	0.399	-0.133 (0.224)	0.554	-0.133 (0.157)	0.396
Year of transplant								
1999 - 2002 (ref)	-	0.030	-	0.053	-	0.036	-	0.061
2003 - 2006	0.075 (0.097)	0.442	0.074 (0.103)	0.474	0.073 (0.098)	0.453	0.073 (0.103)	0.480
2007 - 2011	-0.109 (0.110)	0.319	-0.110 (0.106)	0.297	-0.106 (0.110)	0.333	-0.106 (0.106)	0.314
Graft type								
Peripheral blood (ref)	-	< 0.001	-	< 0.001	-	< 0.001	-	< 0.001
Bone marrow	-0.387 (0.097)	< 0.001	-0.387 (0.076)	< 0.001	-0.386 (0.097)	< 0.001	-0.386 (0.076)	< 0.001
In vivo T cell depletion								
No(ref)	-	< 0.001	-	< 0.001	-	< 0.001	-	< 0.001
Yes	-0.527 (0.106)	< 0.001	-0.526 (0.086)	< 0.001	-0.526 (0.106)	< 0.001	-0.526 (0.086)	< 0.001
Total body irradiation								
No (ref)	-	0.076	-	0.021	-	0.074	-	0.020
Yes	0.280 (0.158)	0.076	0.280 (0.121)	0.021	0.282 (0.158)	0.074	0.282 (0.121)	0.020

SE is the standard error and * denotes the overall p-value.

6. Conclusion

We have studied when the number of strata is finite, but the number of clusters increases. Zhou et al. (2011) considered highly stratified data which have a large number

Appendices

$$q^{(1)}_{ijk}(u) = \lim_{n_j \to \infty} \frac{1}{n_j} \sum_{i'=1}^{n} \sum_{k'=1}^{n_{i'j}} \int_{s=X_{i'jk'}} \left(Z_{i'jk'} - \frac{s^{(1)}_{COX;j}(\;_0; s)}{s^{(0)}_{COX;j}(\;_0; s)} \right)$$

$$w^{COX}_{i'jk'}(s) \frac{\exp(\;_0' Z_{i'jk'}) I(u \quad s)}{s^{(0)}_{Cj}(\;_0; u)} dM^{1}_{i'jk'}$$

C9.9626 Tf 4.469 1.495g32 9.96321 6.9738 Tf -1.07f 2.819 1.992 Td [(0)]TJ/F21 6.9738 Tf 2.694 -1.992 Td.40T

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Supplementary Materials for Competing risks regression for clustered data with covariate-dependent censoring

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ABSTRACT
We provide proofs for Theorems 1 to 3. We also provide the asymptotic variances of the proposed estimators for and $_{10j}(t)$ when using Kaplan-Meier estimates for censoring.

1. Supplementary Materials

1.1. Proof of Theorem 1

Using similar arguments as in Kim et al. (2020), we have

$$
\begin{aligned}
&\hat{G}_{Cj}(t \mid Z_{ijk}) \quad G_{Cj}(t \mid Z_{ijk}) \\
= &\frac{G_{Cj}(t \mid Z_{ijk})}{n_j}\int_{u=0}\sum_{i'=1}^{n}\sum_{k'=1}^{n_{i'j}} \frac{e^{\gamma_0^{|} Z_{ijk}} I(u \quad t)}{S_{Cj}^{(r)}(\ _0;u)} dM^{C}_{i'jk'}(u) \\
&\frac{G_{Cj}(t \mid Z_{ijk})}{n_j}\int_{u=0} h^{|}_{Cj}(t;0;Z_{ijk}) \quad {}_{C}^{1}\sum_{i'=1}^{n}\sum_{j'=1}^{S} {}_{i'j'} \\
&\sum_{k'=1}^{n_{i'j'}}\left(Z_{i'j'k'} \quad \frac{S_{Cj'}^{(1)}(\ _{0C};s)}{S_{Cj'}^{(0)}(\ _{0C};s)}\right) dM^{C}_{i'j'k'}(u) + o_p(n_j^{1=2});
\end{aligned} \tag{1}
$$

where

$$
h_{Cj}(t;u;Z) = e^{\gamma_0^{|} Z}\int_{s=u}^{t}\left(Z \quad \frac{S_{Cj}^{(1)}(\ _0;s)}{S_{Cj}^{(0)}(\ _0;s)}\right) d\ _{C0j}(s):
$$

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Using (1), we have

$$\frac{\hat{G}_{Cj}(t \mid Z_{ijk})}{\hat{G}_{Cj}(X_{ijk} \wedge t \mid Z_{ijk})} - \frac{G_{Cj}(t \mid Z_{ijk})}{G_{Cj}(X_{ijk} \wedge t \mid Z_{ijk})}$$

$$= I(X_{ijk} < t)\frac{G_{Cj}(X_{ijk} \mid Z_{ijk})\{\hat{G}_{Cj}(t \mid Z_{ijk}) - G_{Cj}(t \mid Z_{ijk})\}}{\hat{G}_{Cj}(X_{ijk} \mid Z_{ijk})G_{Cj}(X_{ijk} \mid Z_{ijk})} - \frac{G_{Cj}(t \mid Z_{ijk})\{\hat{G}_{Cj}(X_{ijk} \mid Z_{ijk}) - G_{Cj}(X_{ijk} \mid Z_{ijk})\}}{\hat{G}_{Cj}(X_{ijk} \mid Z_{ijk})G_{Cj}(X_{ijk} \mid Z_{ijk})}$$

$$= -I(X_{ijk} < t)\frac{G_{Cj}(t \mid Z_{ijk})}{G_{Cj}(X_{ijk} \mid Z_{ijk})}\left(\frac{1}{n_j}\sum_{i'=1}^{n}\sum_{k'=1}^{n_{i'j}}\int_{s=0} \exp(\gamma_0^T Z_{ijk})\frac{I(X_{ijk} < s \le t)}{s^{(0)}_{Cj}(\gamma_0; s)}dM^C_{i'jk'}(s) + \frac{1}{n_j}\sum_{i'=1}^{n}\sum_{j'=1}^{J}\sum_{k'=1}^{n_{i'j'}}\int_{s=0} h^{\gamma}_{Cj}(t; X_{ijk}; Z_{ijk})^T\,\Omega_C^{-1}\left\{Z_{i'j'k'} - \frac{s^{(1)}_{Cj'}(\gamma_0; s)}{s^{(0)}_{Cj'}(\gamma_0; s)}\right\}dM^C_{i'j'k'}(s)\right) + o_p(n_j^{-1/2}).$$

Hence,

$$\hat{w}^{COX}_{ijk}(t) - w^{COX}_{ijk}(t)$$

$$= -I(C_{ijk} \ge T_{ijk} \wedge t)I(X_{ijk} < t)\frac{G_{Cj}(t \mid Z_{ijk})}{G_{Cj}(X_{ijk} \mid Z_{ijk})}\left(\frac{1}{n_j}\sum_{i'=1}^{n}\sum_{k'=1}^{n_{i'j}}\int_{s=0} \frac{\exp(\gamma_0^T Z_{ijk})I(X_{ijk} < s \le t)}{\cdots}\right.$$

$$
\begin{aligned}
& U_{COX}(\ _0) \\
&= \sum_{i=1}^{n} \left[\sum_{j=1}^{S} \sum_{k=1}^{n_{ij}} \int_0 \left(Z_{ijk} - \frac{S^{(1)}_{COX;j}(\ _0;u)}{S^{(0)}_{COX;j}(\ _0;u)} \right) \hat{w}^{COX}_{ijk}(u)dM^{1}_{ijk}(u) \right] \\
&= \sum_{i=1}^{n} \sum_{j=1}^{S} \sum_{k=1}^{n_{ij}} \int_0 \left(Z_{ijk} - \frac{S^{(1)}_{COX;j}(\ _0;u)}{S^{(0)}_{COX;j}(\ _0;u)} \right) w^{COX}_{ijk}(u)dM^{1}_{ijk}(u) \\
&+ \sum_{i=1}^{n} \sum_{j=1}^{S} \sum_{k=1}^{n_{ij}} \int_0 \left(Z_{ijk} - \frac{S^{(1)}_{COX;j}(\ _0;u)}{S^{(0)}_{COX;j}(\ _0;u)} \right) \left(\hat{w}^{COX}_{ijk}(u) - w^{COX}_{ijk}(u) \right) dM^{1}_{ijk}(u) \qquad (3) \\
&= \sum_{i=1}^{n} \sum_{j=1}^{S} \sum_{k=1}^{n_{ij}} \int_0 \left(Z_{ijk} - \frac{s^{(1)}_{COX;j}(\ _0;u)}{s^{(0)}_{COX;j}(\ _0;u)} \right) w^{COX}_{ijk}(u)dM^{1}_{ijk}(u) \\
&+ \sum_{i=1}^{n} \sum_{j=1}^{S} \sum_{k=1}^{n_{ij}} \int_0 \left(Z_{ijk} - \frac{S^{(1)}_{COX;j}(\ _0;u)}{S^{(0)}_{COX;j}(\ _0;u)} \right) \left(\hat{w}^{COX}_{ijk}(u) - w^{COX}_{ijk}(u) \right) dM^{1}_{ijk}(u) \\
&+ o_P(n^{1=2}):
\end{aligned}
$$

Using (2), the second term of the last equation of (3) is asymptotically equivalent to

$$
\sum_{i=1}^{n} \sum_{j=1}^{S} \sum_{k=1}^{n_{ij}} \int_0 q^{(1)}_{ijk}(t)dM^{C}_{ijk}(t);
$$

where

$$
\begin{aligned}
q^{(1)}_{ijk}(u) = & \lim_{n_j \to \infty} \frac{1}{n_j} \sum_{i'=1}^{n} \sum_{k'=1}^{n_{i'j}} \int_{s=X_{i'jk'}} \left(Z_{i'jk'} - \frac{s^{(1)}_{COX;j}(\ _0;s)}{s^{(0)}_{COX;j}(\ _0;s)} \right) \\
& w^{COX}_{i'jk'}(s) \frac{\exp(\ _0^{\,|} Z_{i'jk'}) I(u \ \ s)}{s^{(0)}_{Cj}(\ _0;u)} dM^{1}_{i'jk'}(s) \\
& + \frac{1}{n} \sum_{i'=1}^{n} \sum_{j'=1}^{S} \sum_{k'=1}^{n_{i'j'}} \int_{s=X_{i'j'k'}} \left(Z_{i'j'k'} - \frac{s^{(1)}_{COX;j}(\ _0;s)}{s^{(0)}_{COX;j}(\ _0;s)} \right) w^{COX}_{i'j'k'}(s) \\
& h^{|}_{i'j'k'}
\end{aligned}
$$

Since $^{cox}_{i::}$ and $^{cox}_{i::}$

where

$\phi^{(1)}$

Plugging (6) into (5),

$$p \overline{n_j} \; n \; b_{10j}(t) \qquad {}_{10j}(t$$

Using (9) and (11),

$$\sqrt{n_j}\left\{ b_{10j}(t) \quad {}_{10j}(t) \right\} = \frac{1}{\sqrt{n_j}} \left\{ \sum_{i=1}^{n} W^{(1)}_{ij:}(t) + W^{(2)}_{ij:}(t) \right\} + o_P(1); \qquad (12)$$

where

$$W^{(1)}_{ij:}(t) = \quad {}_j(\quad {}^{COX}_{i::} + \quad {}^{COX}_{i::})^{|} f \; (\quad {}_0) g^{\;1} \int_0^t \frac{s^{(1)}_{COX;j}(\quad {}_0; s)}{s^{(0)}_{COX;j}(\quad {}_0; s)}$$

Consider the first term of (13) and using (12),

$$\sqrt{n_j}\,\exp(\ {}^{|}_{0} Z_0)\,\{\,b_{10j}(t) \quad {}_{10j}(t)\}$$

$$= \frac{1}{\sqrt{n_j}}\exp(\ {}^{|} Z_0)\left(\sum_{i=1}^{n} W^{(1)}_{ij:}(t) + W^{(2)}_{ij:}(t)\right) + o_P(1):$$

Now consider the second term of (13) and using the functional delta method,

$$\sqrt{n_j}\,\{\,b_{10j}(t)\exp(b^{|} Z_0) \quad b_{10j}(t)\exp(\ {}^{|}_{0} Z_0)\}$$

$$= \sqrt{n_j}\,b_{10j}(t)\,\{\exp(b^{|} Z_0) \quad \exp(\ {}^{|}_{0} Z_0)\}$$

$$= \sqrt{n_j}\,b_{10j}(t)\exp(\ {}^{|} Z_0) Z^{|}_{0}\ \{)$$ exp776 Tf 6.398 -.

The variance can be estimated by

$$\hat{\sigma}^{KM}_{10j}(t) = \frac{1}{n_j}\sum_{i=1}^{n_j}\left\{\hat{W}^{(1)}_{KM;ij\cdot}(t) + \hat{W}^{(2)}_{KM;ij\cdot}(t)\right\}^2;$$

where

$$\hat{W}^{(1)}_{KM;ij\cdot}(t) = \xi_j(\hat{\beta}^{KM}_{i\cdot\cdot} + \hat{\beta}^{KM}_{i\cdot\cdot})^{\top}(I_{KM}(\hat{\beta}^{KM})/n)^{-1}\int_0^t \left(\frac{S^{(1)}_{KM;j}(\hat{\beta}^{KM};s)}{S^{(0)}_{KM;j}(\hat{\beta}^{KM};s)}\right) d\hat{\Lambda}^{KM}_{10j}(s);$$

$$\hat{W}^{(2)}_{KM;ij\cdot}(t) = \xi_{ij}\sum_{k=1}^{n_{ij}}\left(\int_0^t \frac{\hat{w}^{KM}_{ijk}(s)\,d\hat{M}^{1}_{KM;ijk}(s)}{S^{(0)}_{KM;j}(\hat{\beta}^{KM};s)}\right.$$

$$\int_0^t \hat{q}^{(2)}(s;t)\,d\hat{M}^{c}_{KM;ijk}(s);$$

$\hat{q}^{(2)}_{KM}(s;t)$

$\hat{W}^{(2)}_{KM}$ -1.2 -71.93 [(K)-69(M -1. 93 Td [(K)9J088ss;)-371(t)]T -1.261 T01)s