

$\bar{t}_i = \frac{1}{n} \sum_{j=1}^n t_{ij}$

$p_i = P\{T_i > |t_i| \mid \bar{t}_i = \mu_i\}$

$E_i = E\{T_i \mid \bar{t}_i = \mu_i\}$

$\{T_i > \gamma/\mu_i\}$

$$PCE = P E_i .$$

$\bar{t}_i = \frac{1}{n} \sum_{j=1}^n t_{ij}$

$p_i = P\{T_i > \gamma/\mu_i \mid \bar{t}_i = \mu_i\}$

$E_i = E\{T_i \mid \bar{t}_i = \mu_i\}$

$$FWE = P_i E_i .$$

$FWE = PCE$

$\bar{t}_i = \frac{1}{n} \sum_{j=1}^n t_{ij}$

$p_i = P\{T_i > \gamma/\mu_i \mid \bar{t}_i = \mu_i\}$

$E_i = E\{T_i \mid \bar{t}_i = \mu_i\}$

$V = \frac{1}{n} \sum_{j=1}^n t_{ij}^2$

	U	V	m
	T	S	m
	Y	R	m

$$FDR = E[V/R],$$

$$FWE = P[V > \alpha].$$

$$FDR = P[R > \alpha] = P[V > \alpha] = FWE.$$

$$FDR = P[V > \alpha] = FWE.$$

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$$FDR = P[V > \alpha] = P[V > \alpha] = FWE.$$

F

$$FWE = P[E_1 \cup E_2 \cup \dots \cup E_m] = P[E_1] + P[E_2] + \dots + P[E_m] - P[E_1 \cap E_2] - \dots + P[E_1 \cap E_2 \cap \dots \cap E_m].$$

$$FWE = P[E_1] + P[E_2] + \dots + P[E_m] - m\alpha' + \alpha.$$

$$FWE = P[E_1] + P[E_2] + \dots + P[E_m] - m\alpha' + \alpha.$$

E_i

$[T/\gamma]$

$$FWE = -P [T/\gamma, \dots, T_m/\gamma] - P [T_i/\gamma]$$

α

m

$$P_{RF} p_i = -E d$$

$E d$

$$E d = S \pi^{-(m+)} / W^{-m} p_i^{-m} \cdot -p_i/$$

$E d$

$$p(i) = \frac{i}{m} \frac{q}{\sum_i / i}$$

$$pFDR = E[V/R | R > 0]$$

$$pFDR = p \cdot P(|T_i| > \gamma)$$

$$pFDR = E[V/R | R > 0]$$

$$E_{R > 0} [E[V/R | R > 0]]$$

$$E_{R > 0} [p V / S / V / S] = p$$

$$pFDR = \frac{P(|T_i| > \gamma | \mu_i) \cdot P(\mu_i)}{P(|T_i| > \gamma)}$$

$$pFDR = \frac{F(\gamma/\pi)}{F(\gamma)}$$

$\mathbb{P}(\text{at least } i \text{ hypotheses are rejected}) = \sum_{j=i}^m \binom{m}{j} \alpha^j (1-\alpha)^{m-j}$

This is the cumulative distribution function of a binomial distribution with parameters m and α .

$$\text{pFDR} = \sum_{i=1}^m \mathbb{P}(i \text{ hypotheses are rejected}) \frac{i}{m}$$

$$\text{pFDR} \leq q = \frac{i}{m} q.$$

The above inequality is a direct consequence of the definition of the pFDR and the fact that the number of hypotheses rejected is at most m .

In the above derivation, we assumed that the null hypotheses are rejected with probability α .

This is the case for the Benjamini-Hochberg procedure.

$\mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right] = \mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right]$

$$E \left[\frac{R}{S} \mid \mathcal{F}_T \right] = E \left[\frac{V}{S} \mid \mathcal{F}_T \right]$$

$\mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right] = S \gamma$

$$S \gamma = R - mp_\gamma,$$

$\mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right] = \mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right]$

$$\widehat{FDR}_{YB}(\gamma) = E \left[\frac{R}{S} \mid \mathcal{F}_T \right]$$

$\mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right] = \mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right]$

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$\mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right] = \mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right]$

$$\widehat{FDR}_{YB}(\gamma) = \sum_{b=1}^B \left[\frac{R^b}{S} \mid \mathcal{F}_T \right]$$

$\mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right] = \mathbb{P} \left[\frac{R}{S} \geq \gamma \mid \mathcal{F}_T \right]$

$\hat{FDR}_{YB} p(i) = \frac{p(i)}{r}$

$\hat{FDR}_{YB} p(i) = \alpha$

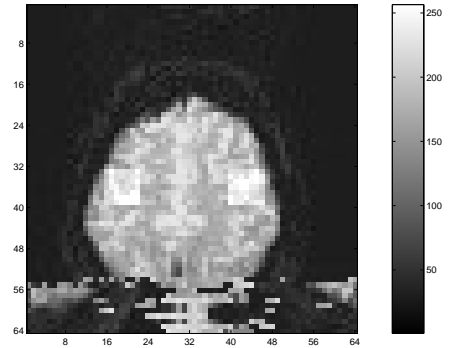
$(1), \dots, (r)$

F

$\hat{FDR}_{YB} p(i) = \frac{p(i)}{r}$

$\hat{FDR}_{YB} p(i) = \alpha$

$(1), \dots, (r)$



Y

X

B

E

$n \times p$

$n \times q$

$q \times p$

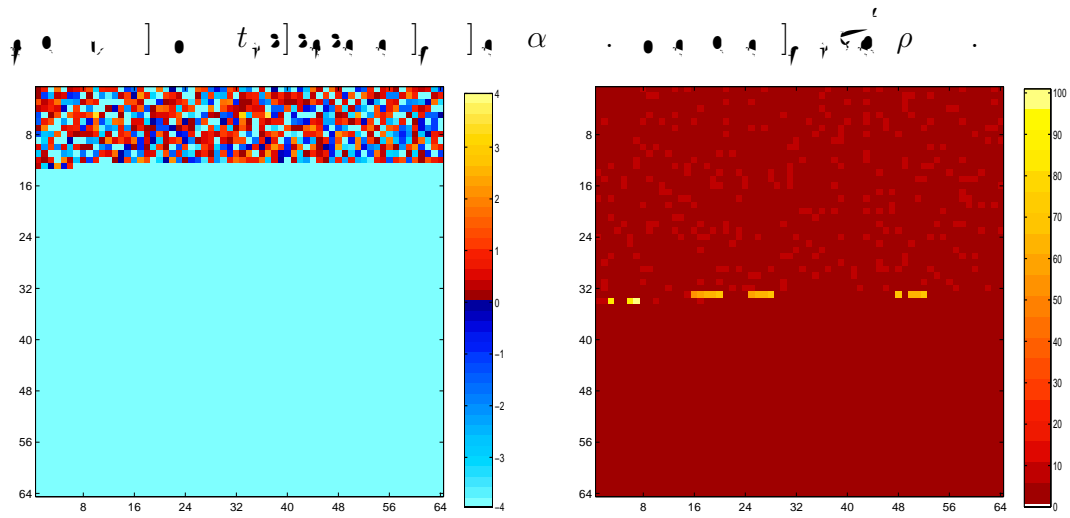
$n \times p$

$n \times p$

$n \times q$

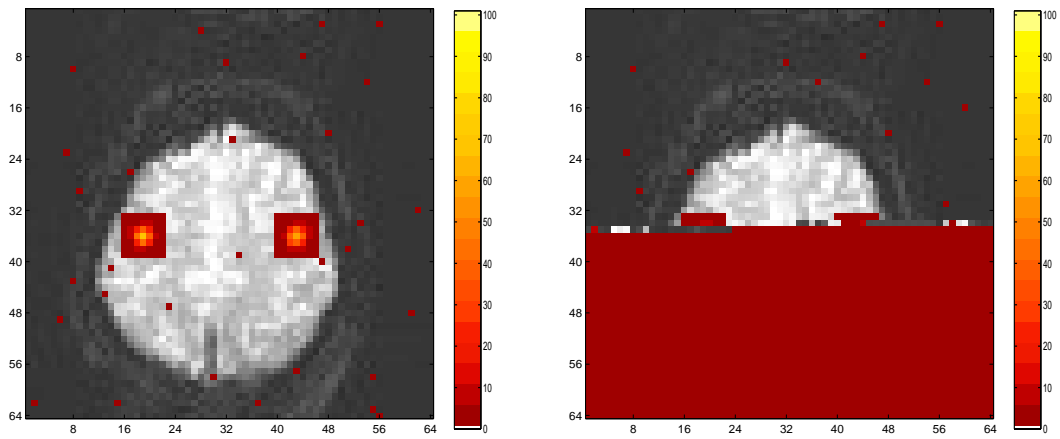
$q \times p$

$n \times p$



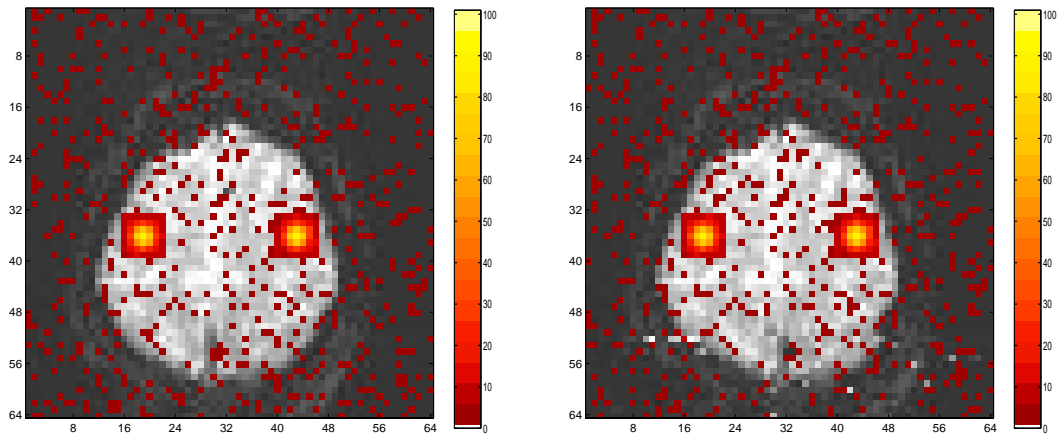
(a) Sample t -statistic image

(b) Unadjusted threshold



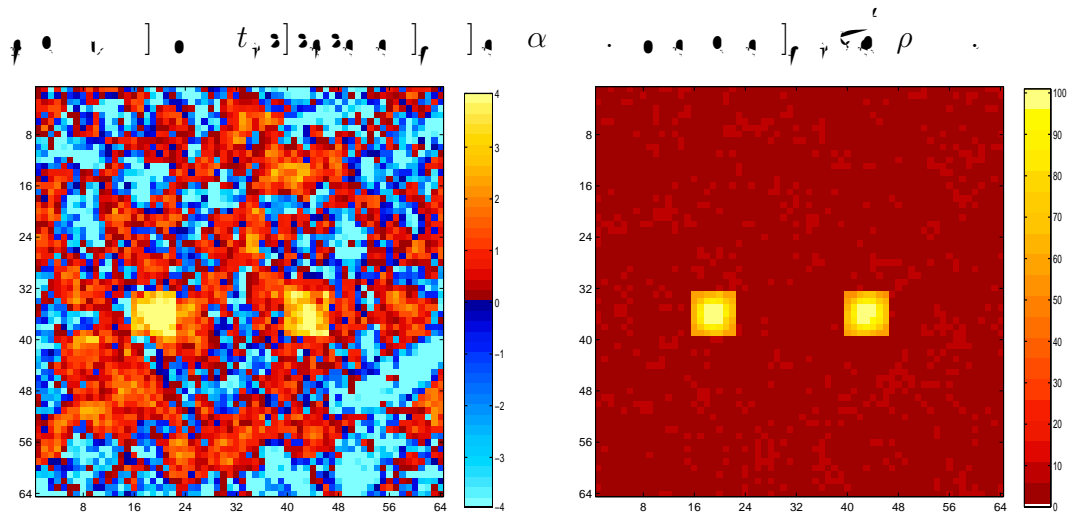
(c) FWE Bonferroni method

(d) FWE Permutation method



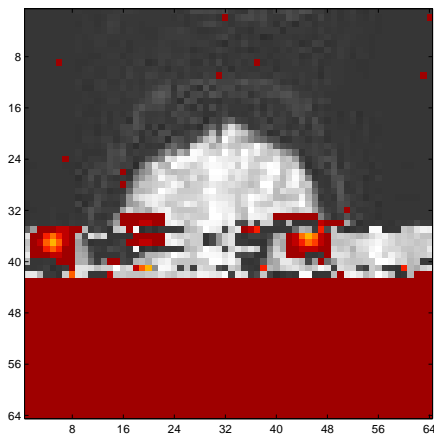
(e) FDR BH method

(f) FDR YB method

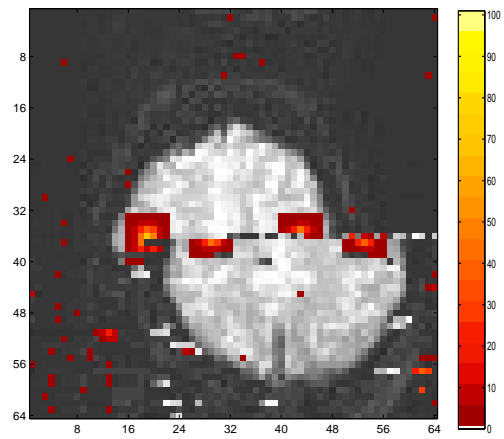


(a) Sample t -statistic image

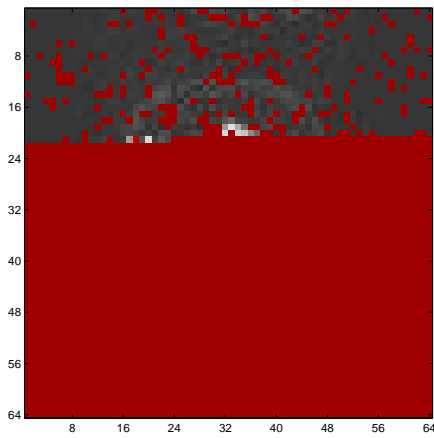
(b) Unadjusted threshold



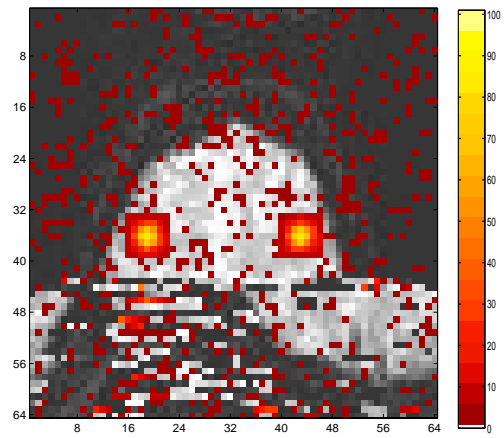
(c) FWE Bonferroni method



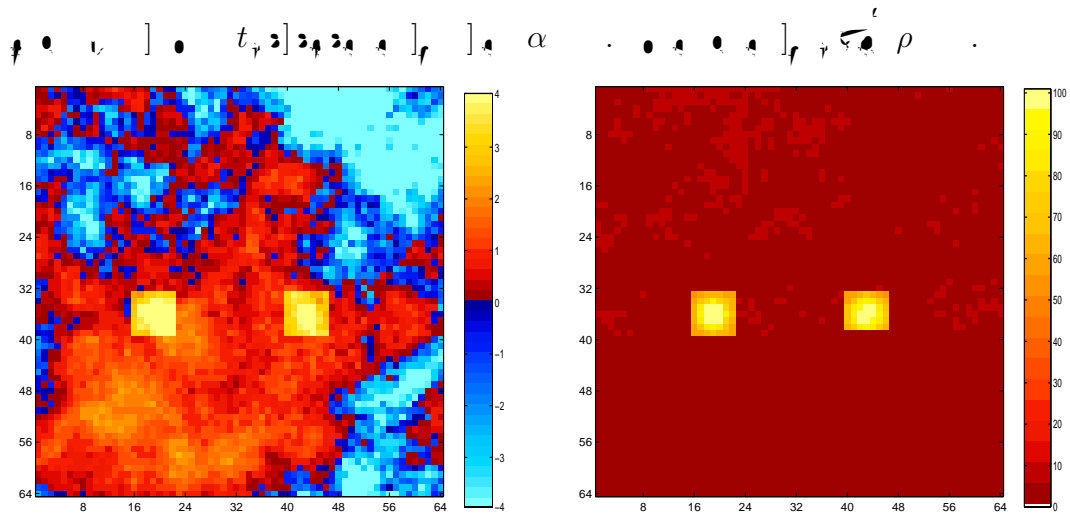
(d) FWE Permutation method



(e) FDR BH method

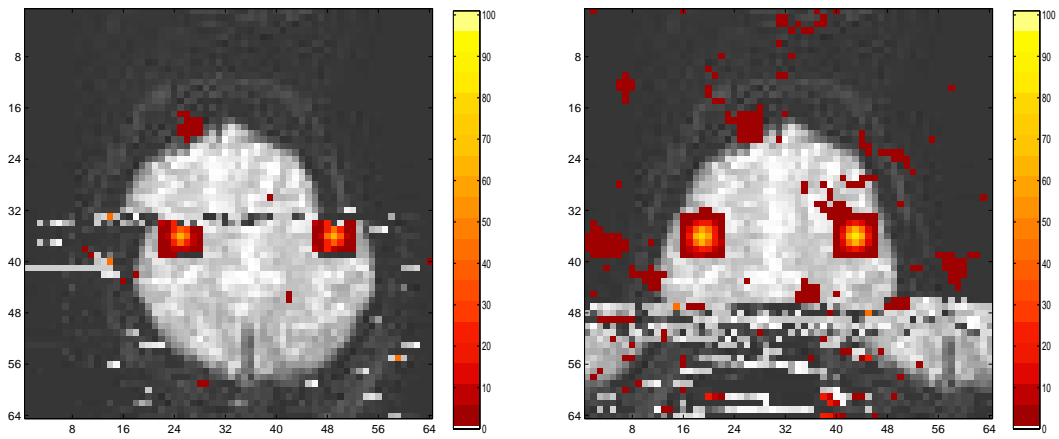


(f) FDR YB method



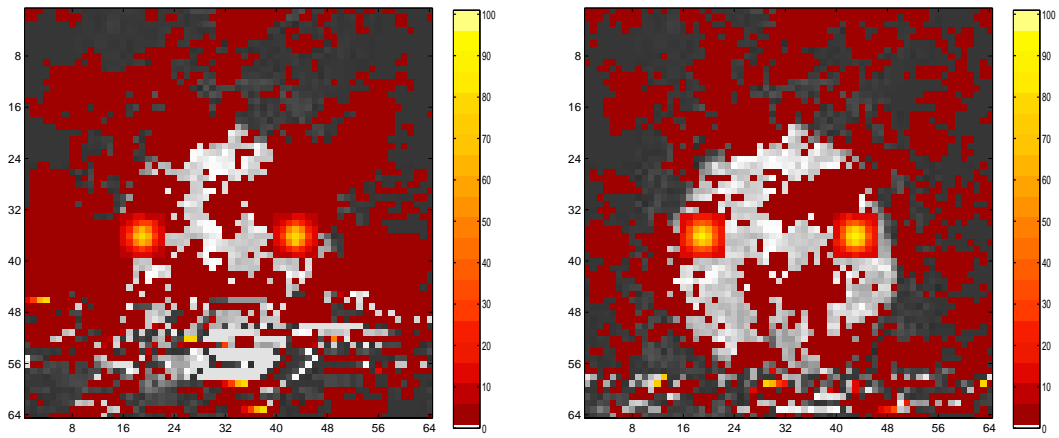
(a) Sample t -statistic image

(b) Unadjusted threshold



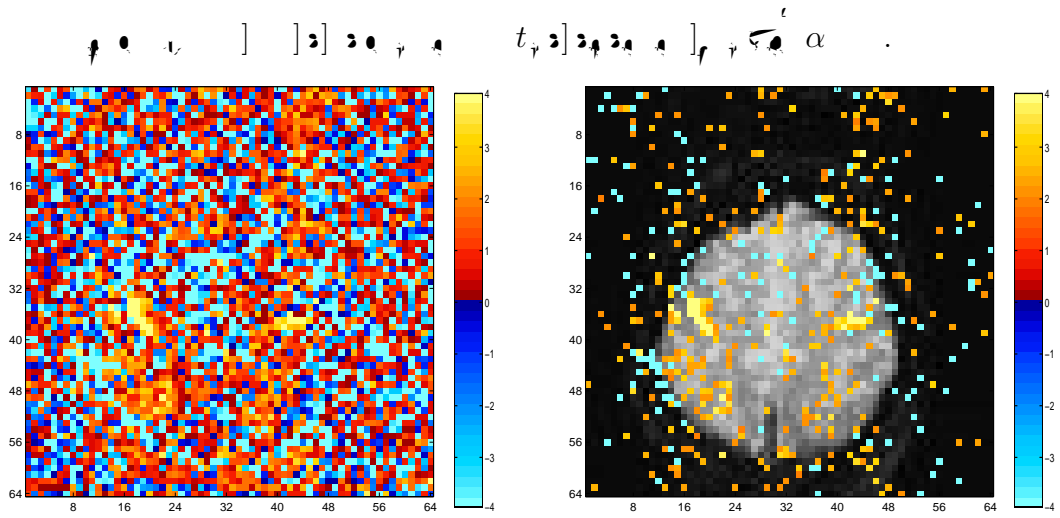
(c) FWE Bonferroni method

(d) FWE Permutation method



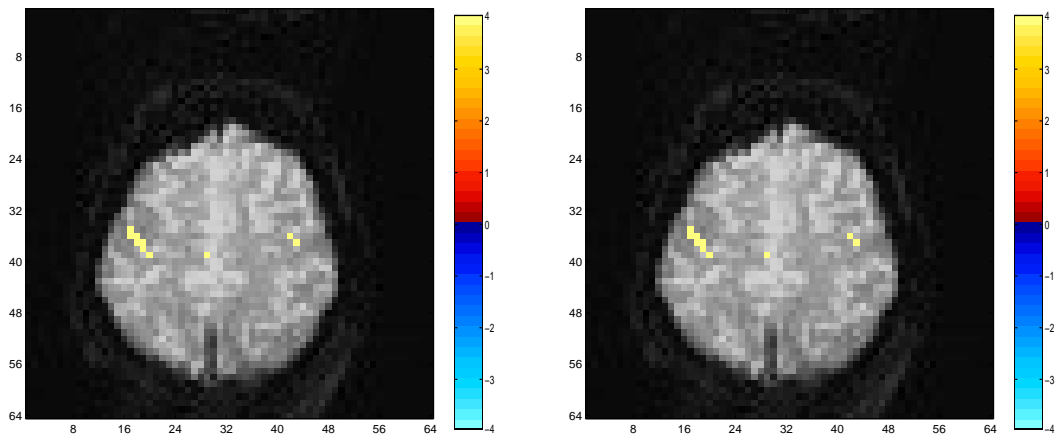
(e) FDR BH method

(f) FDR YB method



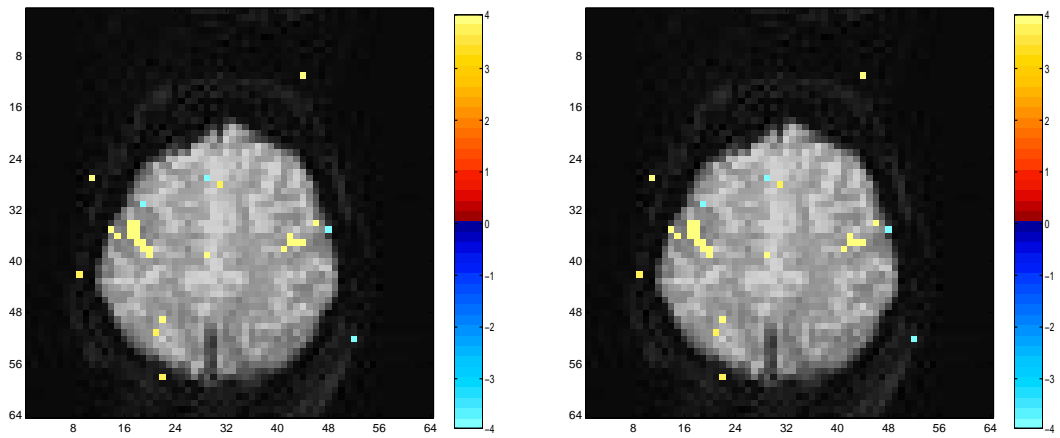
(a) Sample t -statistic image

(b) Unadjusted threshold



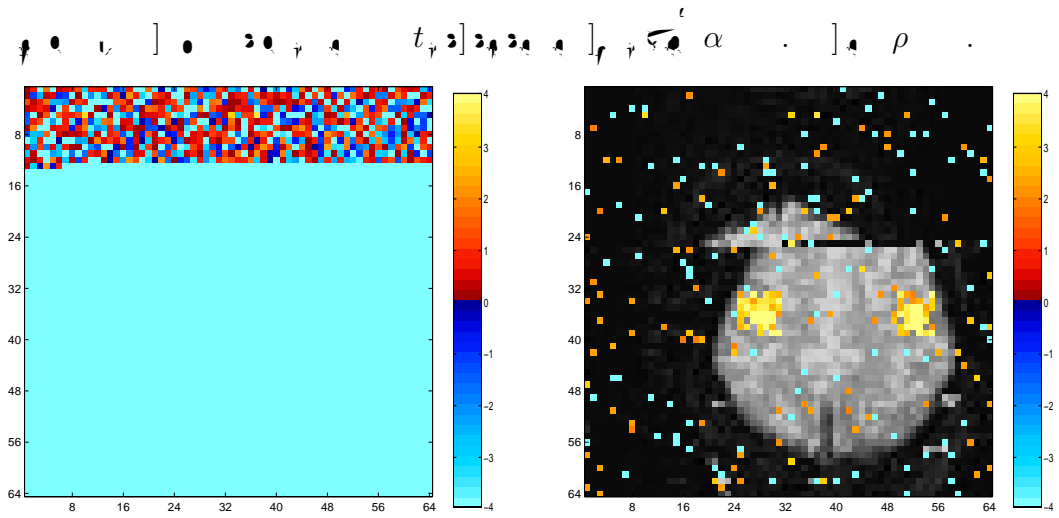
(c) FWE Bonferroni method

(d) FWE Permutation method



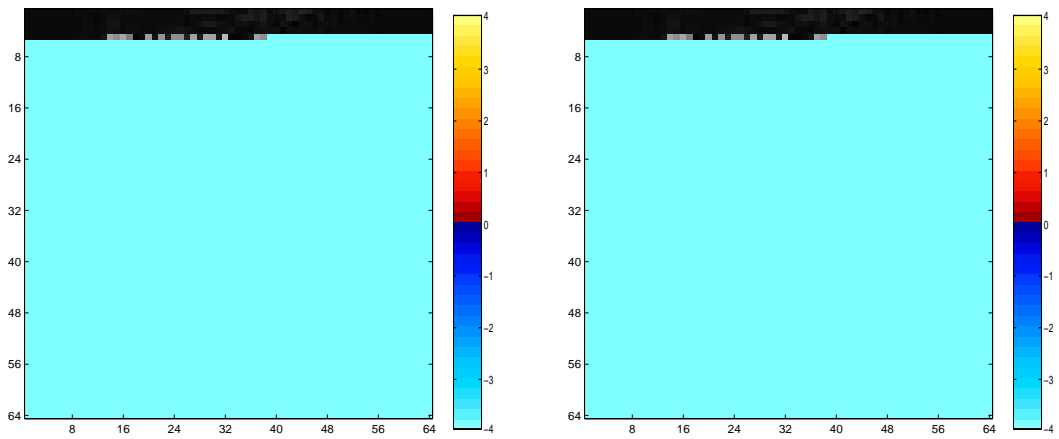
(e) FDR BH method

(f) FDR YB method



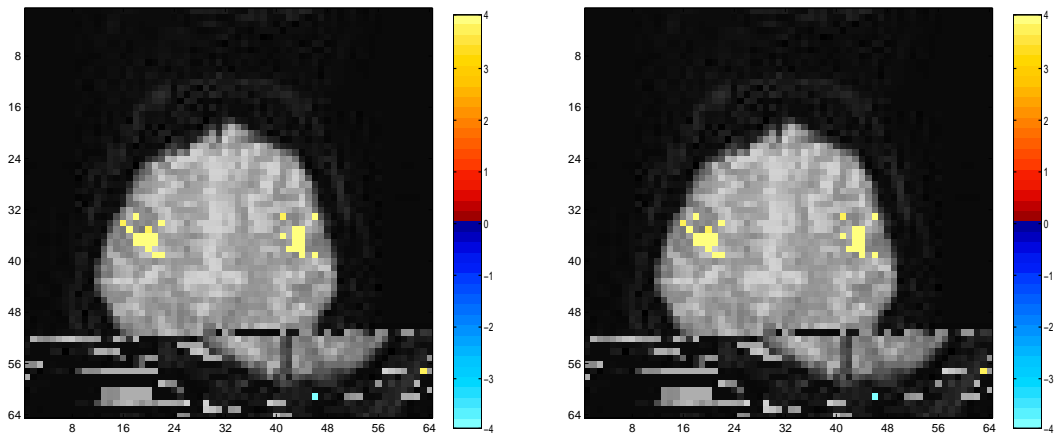
(a) Sample t -statistic image

(b) Unadjusted threshold



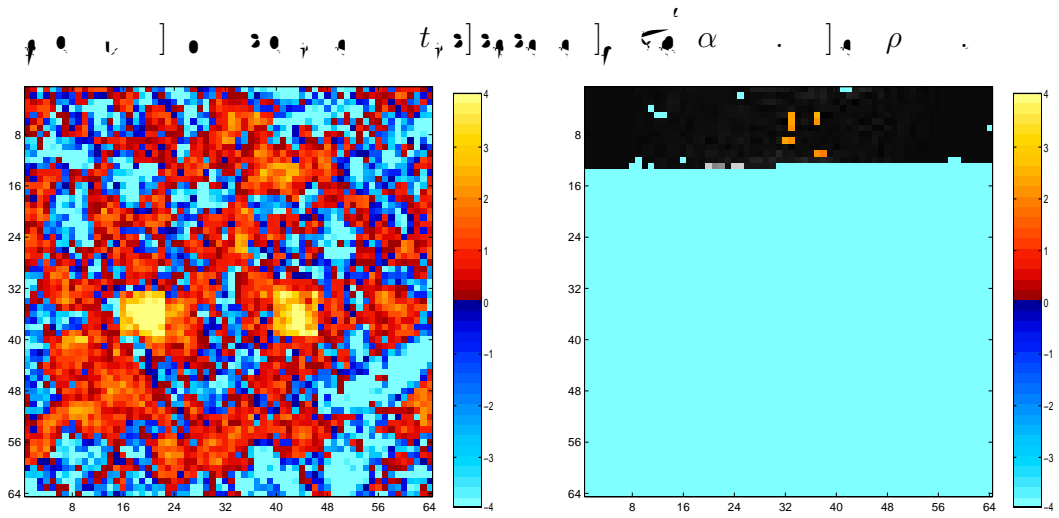
(c) FWE Bonferroni method

(d) FWE Permutation method



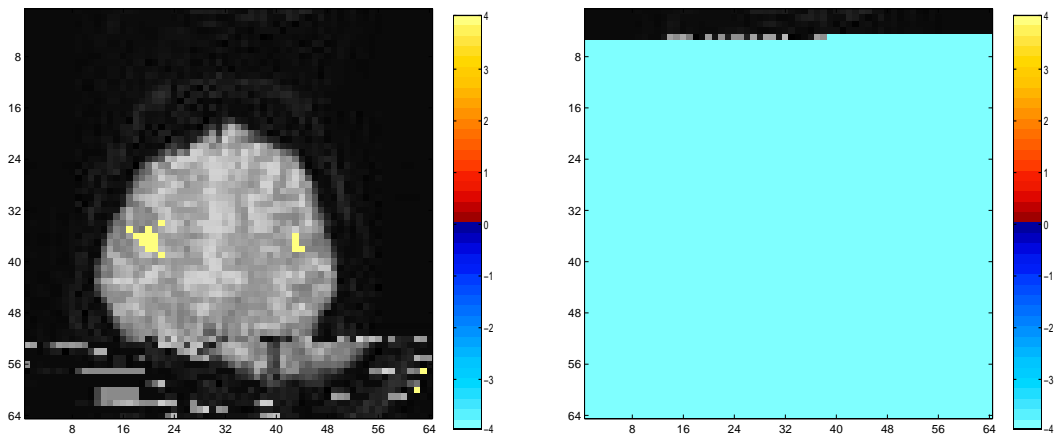
(e) FDR BH method

(f) FDR YB method



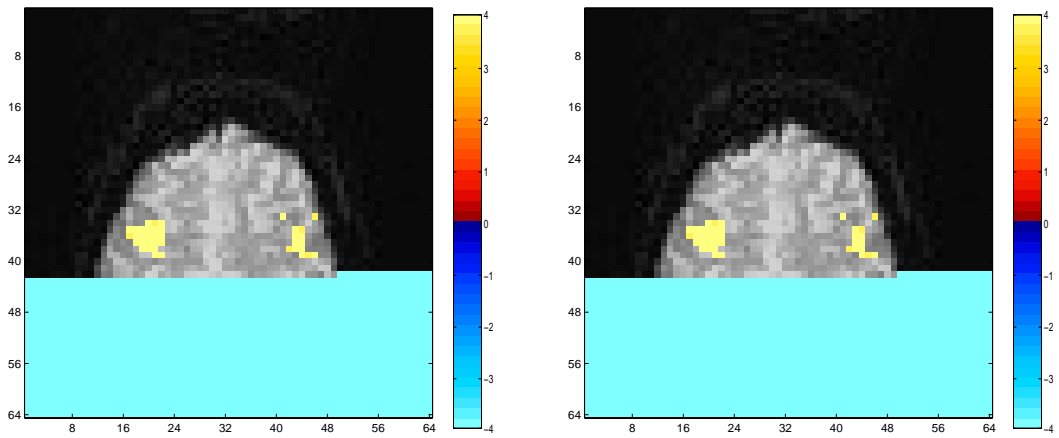
(a) Sample t -statistic image

(b) Unadjusted threshold



(c) FWE Bonferroni method

(d) FWE Permutation method



(e) FDR BH method

(f) FDR YB method

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